DETERMINATION OF THE HEAT TRANSFER COEFFICIENT FROM THE LAWS OF CONSTANT-TEMPERATURE FRONT PROPAGATION

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A method is proposed for determining the heat transfer coefficient from temperature measurements at internal points in bodies of the simplest geometry.

The feasibility of determining the heat transfer coefficient from instantaneous values of the temperature at points inside and at the surface of a body is a matter of continued interest and the problem is solved, for instance, by the well known A. G. Temkin method [1, 2]. In order to avoid determining higher than first-order derivatives of temperature with respect to time, it is found necessary to solve a system of equations after a preliminary solvability analysis, etc. [3].

We will show here another way of determining the heat transfer coefficient, based on laws of the regular heating mode which have been established by this author earlier. In this heating mode the velocity of constant-temperature (T = idem) fronts at Bi = const is:

a) in an infinitely large plate

$$v_T = \left[\frac{\partial (l_0 - x)}{\partial \tau}\right]_T = \frac{a}{l_0}\mu_1 \operatorname{ctg} \mu_1 \frac{x}{l_0}, \quad (1)$$

b) in a cylinder with perfect thermal insulation at the endfaces

$$v_T = \frac{a}{l_0}\mu_1 \frac{J_0\left(\mu_1 \frac{x}{l_0}\right)}{J_1\left(\mu_1 \frac{x}{l_0}\right)}, \quad (2)$$

c) in a sphere

$$v_T = \frac{a}{l_0}\frac{\mu_1^2}{\frac{l_0}{x} - \mu_1 \operatorname{ctg} \mu_1 \frac{x}{l_0}}. \quad (3)$$

We will examine the paths of various constant-temperature (T = idem) fronts in (l_0—x), τ coordinates, on the basis of temperature readings recorded over a period of time at several (at least two) points inside a body. By differentiating a T = idem curve at an arbitrary point x/l_0 within the range of regular heating (segments of various T = idem curves run equidistantly), it is easy to find $v_T = [(\partial(l_0 - x))/\partial\tau]_T$ with l_0 and a known, one can then calculate μ_1 for a given x/l_0 from (1)-(3) and, finally, also the Biot number from the data in [4, 5] relating Bi and μ_1.

The mathematical relations for Bi become much simpler, if velocity $V_T = [(\partial(l_0 - x))/\partial\tau]_T$ is calculated at point $x/l_0 = 1$ on the surface of the given body in a stream. At such a point we have

a) for a plate

$$v_T = \frac{a}{l_0}\mu_1 \operatorname{ctg} \mu_1, \quad (4)$$

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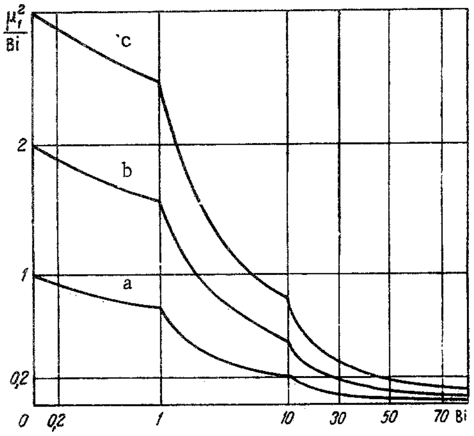

Fig. 1. Curves of μ_1^2/Bi versus Bi: a) for a plate; b) for a cylinder; c) for a sphere.

b) for a cylinder

$$v_T = \frac{a}{l_0}\mu_1\frac{J_0(\mu_1)}{J_1(\mu_1)}, \quad (5)$$

c) for a sphere

$$v_T = \frac{a}{l_0}\cdot\frac{\mu_1^2}{1-\mu_1\operatorname{ctg}\mu_1}. \quad (6)$$

It is well known in the theory of heat conduction [4] that μ_1 can be calculated from the following characteristic equations:

a) for a plate

$$\mathrm{Bi} = \frac{\mu_1}{\operatorname{ctg}\mu_1}, \quad (7)$$

b) for a cylinder

$$\mathrm{Bi} = \mu_1\frac{J_1(\mu_1)}{J_0(\mu_1)}, \quad (8)$$

c)

$$\mathrm{Bi} = 1-\mu_1\operatorname{ctg}\mu_1. \quad (9)$$

A comparison between Eqs. (4)-(6) and Eqs. (7)-(9) leads to the following relation for the dimensionless velocity $\bar{v}_T = [(\partial(1-(x/l_0)))/\partial\mathrm{Fo}]_T$ at the outer boundary, applicable to all three kinds of configurations considered here:

$$\bar{v}_T = v_T\frac{l_0}{a} = \frac{\mu_1^2}{\mathrm{Bi}}. \quad (10)$$

In Fig. 1 are shown curves which represent relation (10) in $\bar{v}_T$, Bi ($v_T = \mu_1^2/\mathrm{Bi}$) coordinates for a plate (curve a), for a cylinder (curve b), and for a sphere (curve c).

Differentiating the curves of T = idem front paths at points $x/l_0 = 1$ on the body surface within the range of regular heating, when these curves remain equidistant beginning at the outer boundary, will yield $v_T = (a/l_0)\cdot(\mu_1^2/\mathrm{Bi})$. We then calculate $\bar{v}_T = v_T(l_0/a) = \mu_1^2/\mathrm{Bi}$ and determine Bi from the graph.

We note that in this method of determining the heat transfer coefficient the quantity μ_1^2, which is very important in heat calculations, acquires the meaning of a modified dimensionless velocity $\bar{v}_T\mathrm{Bi}$ of constant-temperature fronts at the outer surface of a body totally within the range of regular heating.

We note, in conclusion, that this method applies to a heat transfer coefficient which remains constant with time. This method does not require that the ambient medium around the body be known and this, in addition to the simplicity of calculations, dictates its choice for practical applications.

NOTATION

$T(x, \tau)$	is the instantaneous temperature at a point of a body;
l_0	is the characteristic dimension of a body (half-thickness of a plate, radius of a cylinder, or radius of a sphere);
x	is the dimensional space coordinate;
x/l_0	is the dimensionless space coordinate;
τ	is time;
$\mathrm{Fo} = a\tau/l_0^2$	is the Fourier number;
a	is the thermal diffusivity;
Bi	is the Biot number;
μ_1	is the first root of the characteristic equations of heat conduction;
v_T	is the dimensional velocity of constant-temperature fronts;
$\bar{v}_T$	is the dimensionless velocity of constant-temperature fronts.

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